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ANALYTICAL ANALYSIS FOR SIMPLY SUPPORTED COMPOSITE PLATES UNDER UNIFORMLY DISTRIBUTED LOAD

JAWAD K. ZEBOON

Southern Technical University, Basra, Iraq

ABSTRACT

The response of simply supported cross-ply symmetrically composite plates subjected to uniformly distributed load, with lamination [0 90 90 0] discussed. Matlab is used to perform the analysis, depending on classical lamination plate theory. A number of factors such as aspect ratio, side to thickness ratio and modulus ratio and their effect on deflection and stresses of laminated composite plate, subjected to a uniformly distributed load have been studied. The results showed that, the effect of coupling is to increase the deflection to increase the aspect ratio and modulus ratio, and increase the stress with an increase in the side to thickness ratio and modulus ratio.

KEYWORDS: Nanocomposite Plates, Fiber, Versus Modulus Ratio

INTRODUCTION

Composite materials are used in different systems such as space structures, automobiles, sport equipment and electronic circuit boards. These materials are efficiently in applications, that required high strength to weight and stiffness to weight ratios.

Fiber- reinforced composites are manufactured in laminates. This laminate consists of individual lamina or plies. There are many factors that effect on laminate response to applied loads. Fiber material and orientation, fiber and matrix material and layer sequence are some of the variables, affecting the response of a laminate.

Analysis of deformation in composite structure is of fundamental importance, in the experimental determination of the layer properties and exact solutions are useful in developing a numerical model.

Xu and Wu [1] explained a two dimensional analytical solution for simply supported composite beams with interlayer slips by consideration of a shear deformation effect. Shen [2] gave the nonlinear analysis for bending of simply supported functionally graded nanocomposite plates, subjected to transverse uniform or sinusoidal loads in thermal environments and for this purpose, he used a higher order theory to derive governing equations.

Torabizadeh and Fereidoon [3] presented an analytical and numerical solution for general laminated and thermal loading, based on classical lamination plate theory (CLPT). Sidda, Ramanjancy, Suresh and Vijay used a finite element analysis, to study the effect of transverse shear deformation on deflection and stress of laminated composite plates. Marina [4] presented the equations of bending in an arbitrary cross section of simply supported plates, under distributed load based on a partial layer wise theory, which is based on the assumed displacement field in the form of the double trigonometric Fourier's series.

Junaid, Agarwal and Vikas [5] studied the behavior of FSDT plates under transverse loading condition and estimated the influence of stacking sequence, fiber orientation, layer thickness, aspect ratio and the number of layers in the laminated composite plates on the prediction of maximum strength of the plates.

The objective of this investigation is to investigate the response of simply supported composite plates subjected to uniformly distributed load.

The Problem Statement

A rectangular composite plate with length a, width b and thickness h, consisted of 4 plies was considered. Ply orientation were in 0 and 90 degrees and the stacking sequence of the laminate was cross-plied only as in Figure (1)

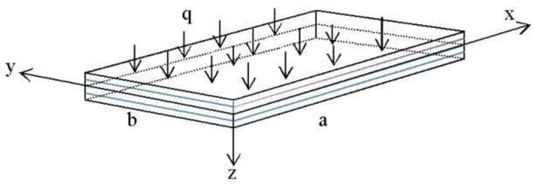

Figure 1: Schematic View of Composite Plate

THEORETICAL FORMULATION

Governing Equation

The governing equation for static bending in the absence of thermal effect and in plane forces is [6].

$$D_{11}\frac{\partial^4 w}{\partial x^4} + 2(D_{12} + 2D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 w}{\partial y^4} = q \qquad 1$$

Where D (Bending stiffness)

W (Displacement along the coordinate-z-)

q (The applied load)

The simply supported boundary conditions on all four edges of the rectangular plates are.

$$w = 0 \text{ at } x = 0\,, a \text{ and } y = 0\,, b \qquad 2$$

Where a and b are the dimensions of the plate in x and y respectively.

Deflection function

The displacement w and the applied load $q\ (x, y)$ in the Navier method are expanded in a double trigonometric (Fourier) series of unknown parameters. The simply supported boundary conditions for rectangular plate in equation (2) are satisfied by the following forms of transverse deflection and applied load.

$$w(x, y) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} W_{mn}\, sin\alpha x\ sin\beta y \qquad 3$$

$$q(x, y) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} Q_{mn}\, sin\alpha x\ sin\beta y \qquad 4$$

Where $\alpha = m\pi/a$ and $\beta = n\pi/b$

W_{mn} Are coefficients to be determined such that the governing equation -1- is satisfied everywhere in the domain of the plate.

Q_{mn} Are load coefficient and

$$Q_{mn} = \frac{4}{ab}\int_0^b \int_0^a q(x,y)\ sin\alpha x\ sin\beta y\ dx\ dy \quad 5$$

Substituting of equations -3-,-4 -and -5- in equation -1- results

$$\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\{-W_{mn}[D_{11}\alpha^4 + 2(D_{12} + 2D_{66})\alpha^4\beta^4 + D_{22}\beta^4] + Q_{mn}\}\ sin\alpha x\ sin\beta y = 0 \quad 6$$

The equation above must be satisfied for every point (x,y), $0 < x < a$ and $0 < y < b$, **and** the express inside the brackets should be zero for every m and n.

$$W_{mn} = \frac{Q_{mn}}{D_{11}\alpha^4 + 2(D_{12}+2D_{66})\alpha^4\beta^4 + D_{22}\beta^4} \quad 7$$

Let.

$$D_{11}\alpha^4 + 2(D_{12} + 2D_{66})\alpha^4\beta^4 + D_{22}\beta^4 = d_{mn} \quad 8$$

Therefore $W_{mn} = \frac{Q_{mn}}{d_{mn}}$ **9**

Then the solution in equation -3- becomes

$$w(x,y) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{Q_{mn}}{d_{mn}} sin\alpha x\ sin\beta x \quad 10$$

For uniformly distributed load $q(x,y) = q_o$ by instituting in equation -5- and integrated the Navier solution will be

$Q_{mn} = 0$ for m, n, even **11**

And $Q_{mn} = \frac{16q_o}{\pi^2 mn}$ for m, n, odd **12**

Substituting equation -12- in equation -9-results

$$W_{mn} = \frac{16q_o}{\pi^2 mn d_{mn}} \text{ for } m, n \text{ odd} \quad 13$$

And therefore the displacement in simply supported composite plate under uniformly distributed load will be

$$w(x,y) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{16q_o}{\pi^2 mn d_{mn}} sin\alpha x\ sin\beta x \quad 14$$

For thin plates the in-plane stresses can be computed from equation [7]

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}^k = -z\begin{Bmatrix}\bar{Q}_{11} & \bar{Q}_{12} & 0\\ \bar{Q}_{12} & \bar{Q}_{22} & 0\\ 0 & 0 & \bar{Q}_{66}\end{Bmatrix}^k \begin{Bmatrix}\frac{\partial^2 w}{\partial x^2}\\ \frac{\partial^2 w}{\partial y^2}\\ 2\frac{\partial^2 w}{\partial x \partial y}\end{Bmatrix} \quad 15$$

By differentiating w and substituting in equation -15- yields

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}^k = z\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{mn}\begin{Bmatrix}(\bar{Q}_{11}^{(K)}\alpha^2 + \bar{Q}_{12}^{(K)}\beta^2)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\\ (\bar{Q}_{12}^{(K)}\alpha^2 + \bar{Q}_{22}^{(K)}\beta^2)\sin\frac{m\pi x}{a}\sin\frac{m\pi y}{b}\\ -2\bar{Q}_{66}^{(K)}\alpha\beta\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b}\end{Bmatrix} \quad 16$$

Where: $\bar{Q}_{ij}^{(k)}$Transformed reduced stiffness matrix.

Validation of the Analysis

In order to verify or validate the theoretical analysis used here a comparison with a numerical example of [8] considered. The boundary condition is simply supported and with the following material and geometric properties

$E_1/E_2 = 25, G = 0.5\,E_2,\ \nu_{12} = 0.3,\ a/b = 1\ ,\ a/h = 10$

The results are introduced here in non-dimensional form using the following.

$w^- = w * \frac{E_2 h^3}{a^4 q_o}\ ;\ \ \sigma_x^- = \sigma_x\frac{h}{aq_o};\ \ \sigma_y^- = \sigma_y\frac{h}{aq_o}\ ;\ \ \ \tau_{xy}^- = \tau_{xy}\frac{h}{aq_o}$

Table 1 represents a comparison of result of non-dimensional deflection obtained from [8] as a finite element analysis and from the current analysis.

Table 1: Non-Dimensional Displacement

Analysis	w^-
FEM Reference (8)	0.00699
Used analytical analysis	0.0068
Difference (%)	2.7

RESULTS AND DISCUSSIONS

The classical lamination plate theory used here for analysis of simply supported composite plate subjected to uniformly distributed load which is with the width ($b = 10\ m$), thickness ($h = 0.1\ m$) and aspect rai (a/b = **1to 5**) also side to thickness ratio (a/h = **10 to 40**). The applied load q_o was ($1\ N/m^2$). In this analysis 4 plies **(0 90 90 0)**^{0} symmetric composite (Reddy) [9] considered with fowling material properties ($E_2 = 1 * 10^6\ N/m^2$, $\nu_{12} = 0.25$, and $G_{12} = 0.5 * E_2$) and the other properties assumed to be as (E_3=E_2, $G_{13} = G_{12}$, $\nu_{23} = \nu_{13} = \nu_{12} * \frac{E_2}{E_1}$) and different modulus ratios ($E_1/E_2 = 1\ to\ 35$ **).** The results obtained for non dimensional deflections (w^-), normal stresses (σ_x^- , σ_y^-) and shear stresses (τ_{xy}^-) are plotted as a function of aspect ratio (a/b) For different modulus ratios as in Figures 2, 3, 4 and 5 respectively. These figures show the effect of bending-stretching coupling and plate aspect ratio on deflections and stresses.

From Figure 2, it is obvious that non dimensional deflection (w^-) is the maximum at an aspect ratio ($a/b = 1$**),** modulus ratio ($E_1/E_2 = 1$) and minimum at aspect ratio (a/b =5), modulus ratio ($E_1/E_2 = 35$ **).** This behavior is related to young's modulus, where increasing the modulus ratio E_1/E_2causes the coupling coefficient to increase and this causes the nondimensionl deflection (w^-) to decrease. The effect of coupling is signed for a/bless than about 3.5 and insignificant for all values greater than 3.5.

From Figures 3-5 can observe the effect of coupling is to decrease the stress with increasing the aspect ratio,

The normal nondimensional stresses $\bar{\sigma_x}$, $\bar{\sigma_y}$ and shear non dimensional stress $\bar{\tau_{xy}}$ are maximum at $\boldsymbol{a/b}$ **=1** , $\boldsymbol{E_1/E_2 = 35}$ and minimum at $\boldsymbol{a/b = 5}$, $\boldsymbol{E_1/E_2 = 1}$. This is because the area of the composite plate increases as the aspect ratio increase, therefore the applied load per unit area is decreased.

Figures 6-9 show the effect of modulus ratios and the side to thickness ratios on deflection, normal stresses and shear stresses for simply supported composite plates laminate [0 90 90 0], under uniform distributed load. The side to thickness ratio has less effect on the deflection, especially for larger modulus ratios, but has a considerable effect on stresses. The deflection decreases with increasing the modulus ratio and side to thickness ratio, while the stresses decrease with the modulus ratios and increase with increasing the side in thickness.

The nondimensional deflection is maximum for side to thickness ratio ($\boldsymbol{a/h = 35}$**),** modulus ratio ($\boldsymbol{E_1/E_2 = 5}$), and minimum of side to thickness ratio ($\boldsymbol{a/h = 10}$**),** modulus ratio ($\boldsymbol{E_1/E_2 = 53}$) and this can reduce with increasing the plate area with increasing side to thickness ratio, therefore, that can causes the applied load per unit area decreases. The normal stress ($\bar{\sigma_x}$) is maximum at side to thickness ratio ($\boldsymbol{a/b}$ = 35) without considerable effect of the modulus ratio, while ($\bar{\sigma_y}$) and ($\bar{\tau_{xy}}$) is maximum at side to thickness ratio ($\boldsymbol{a/h = 35}$), modulus ratio ($\boldsymbol{E_1/E_2 = 5}$) and minimum of side to thickness ratio ($\boldsymbol{a/h = 10}$), modulus ratio ($\boldsymbol{E_1/E_2 = 35}$).

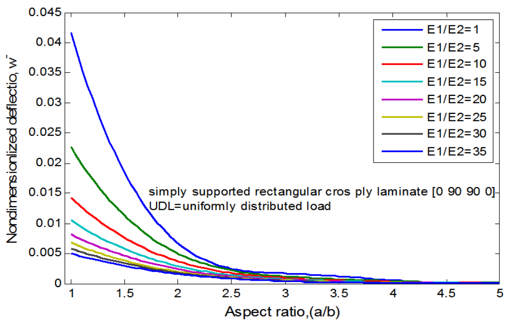

Figure 2: Nondimensionalized Maximum Transverse Deflection (W¯) Versus Aspect Ratio (a/b) For Different Modulus Ratios E1/E2

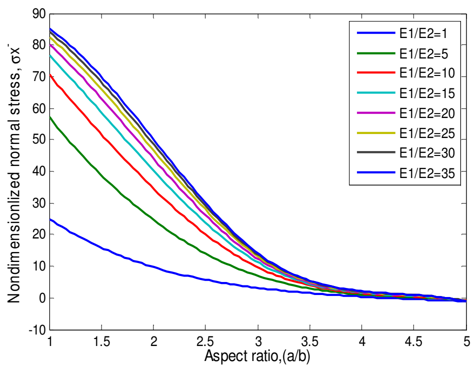

Figure 3: Nondimensionalized Normal Stress (σx¯) Versus Aspect Ratio (a/b) for Different Modulus Ratios E1/E2

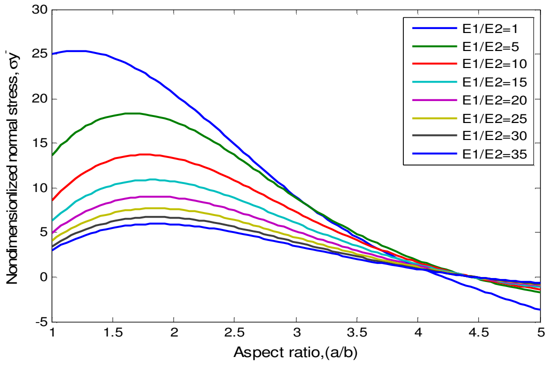

Figure 4: Nondimensionalized Normal Stress (σy¯) Versus Aspect Ratio (a/b) for Different Modulus Ratios E1/E2

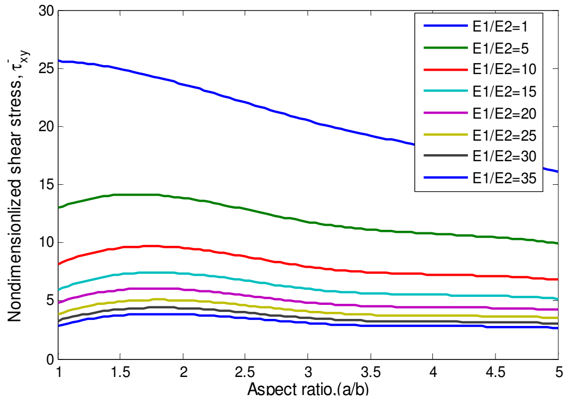

Figure 5: Nondimensionalized Shear Stress (τ^{-}_{xy}) Versus Aspect Ratio (a/b) For Different Modulus Ratios E1/E2

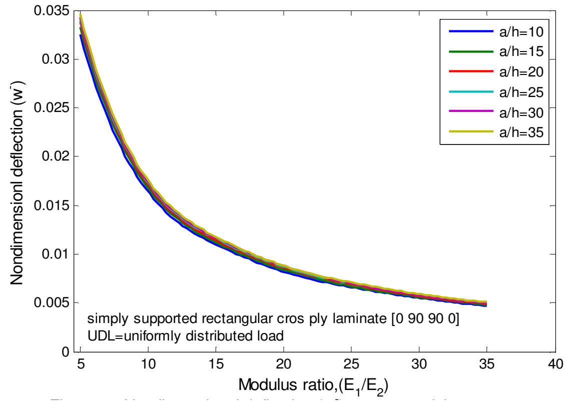

Figure 6: Nondimensionalized Deflection (W¯) Versus Modulus Ratio (E_1/E_2) For Different Side to Thickness Ratios a/h

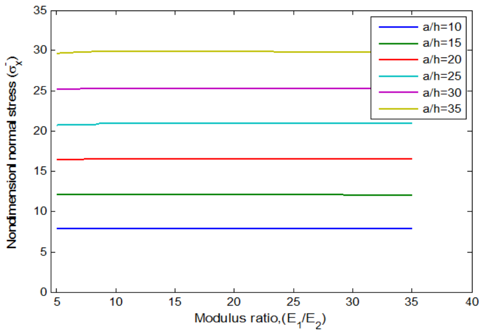

Figure 7: Nondimensionalized Normal Stress ($\bar{\sigma}_y$) Versus Modulus Ratio (E_1/E_2) For Different Side to Thickness Ratios a/h

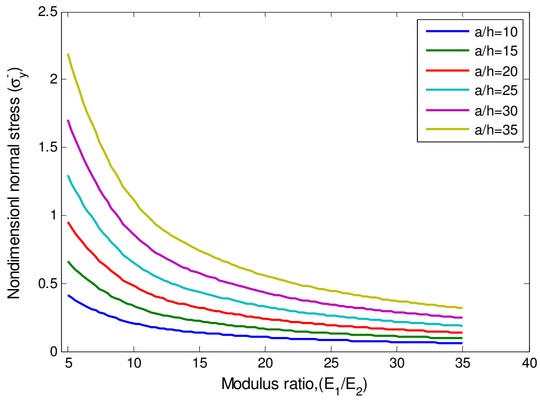

Figure 8: Nondimensionalized Normal Stress ($\bar{\sigma}_y$) Versus Modulus Ratio (E_1/E_2) For Different Side to Thickness Ratios a/h

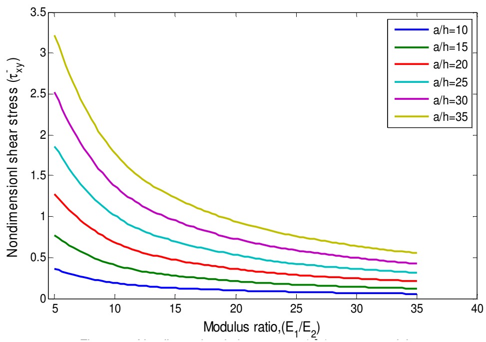

Figure 9: Nondimensional Shear Stress (τ_{xy}) Versus Modulus Ratio (E_1/E_2) For Different Side to Thickness Ratios a/h

CONCLUSIONS

By using a classical lamination plate theory and mat lab code at various aspect ratios, side to thickness ratios and modulus ratios were performed to investigate how it affects the deflections and stresses. The results are plotted in previous figures and observed that, the deflections are larger for small aspect ratios and modulus ratios; the side to thickness ratios has an effect on the deflection for larger ratios of E_1/E_2.

The deflection decrease and the stress increase with increase of modulus ratios and side to thickness ratios.

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